

Om Diagnostic for Dilaton Dark Energy

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Om diagnostic can differentiate between different models of dark energy without the accurate current value of matter density. We apply this geometric diagnostic to dilaton dark energy(DDE) model and differentiate DDE model from LCDM. We also investigate the influence of coupled parameter α on the evolutive behavior of *Om* with respect to redshift z . According to the numerical result of *Om*, we get the current value of equation of state $\omega_{\sigma 0} = -0.952$ which fits the WMAP5+BAO+SN very well.

Keywords: Dark energy; Dilaton; *Om* diagnostic; LCDM.

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1. Introduction

So far, many astronomy observations including SNe Ia[1], SDSS[2], WMAP[3] provide us such a clear outline of the Universe: It is flat and full of an unclumped form of energy density pervading the Universe. The unclumped energy density called "Dark Energy"(DE) with negative pressure, attributes to about 74 percent of the total energy density. The remainder 26 percent of energy density consists of matter including about 22 percent dark matter density and about 4 percent baryon matter density. Beside this, we know little about nature of DE. So, understanding the nature of Dark Energy is one of most challengeable problem for modern astrophysics and cosmology.

As the candidates of DE model, Quintessence[4], Phantom[5], Holographic Dark Energy[6], K-essence[7] and Quintom[8] so on, have been being studied widely by many authors. Of course, the most possible and fundamental candidate of DE is cosmological constant with equation of state(EOS) $\omega = -1$. However, the cosmological constant model suffers from two serious issues: Why the value of cosmological constant Λ is so tiny and not zero which is called "fine-tuning problem". Why the energy density of Λ is just comparable with the matter energy density in recent time which is called "coincidence problem". Alternative to the cosmological constant include scalar field models called Quintessence which have EOS $\omega > -1$, as well as more exotic "phantom" models with EOS $\omega < -1$. The essential characteristics of these dark energy models are contained in the parameter of its equation of state, $p = \omega\rho$, where p and ρ denote the pressure and energy density of dark energy, respectively, and ω is EOS parameter. Quintessence model has been widely studied, and its EOS ω_ϕ , is greater than -1 . Such a model for a broad class of potentials can give the energy density converging to its present value for a wide set of initial conditions in the past and posses tracker behavior. The quintessence potential $V(\phi)$ and the equation of state $\omega_\phi(z)$ may be reconstructed from supernova observations[9].

In our previous papers[10], we have successfully constructed dilaton dark energy(DDE) model where we consider dilaton as a scalar field. For EOS $\omega_\sigma > -1$ DDE model can be regarded as nonminimal coupled Quintessence model while For EOS $\omega_\sigma < -1$ DDE model can be regarded as nonminimal coupled Phantom model. Based on this, we investigated the existence and stability of attractor solutions and obtain that DDE model would admit a late time De sitter attractor solution. Furthermore, parametrization of dark energy function, the influence of dilaton scalar potential on the evolutive behavior of attractor and reconstruction of scalar potential without dependence on model were studied widely by us. In this paper, we will apply a

new geometric method– Om diagnostic to DDE model.

Om , is constructed from the Hubble parameter $H = \frac{\dot{a}}{a}$ determined directly from observational data and provides a *null test* of the LCDM hypothesis. Here $a(t)$ is the scale factor of a Friedmann-Robertson-Walker(FRW) cosmology. In this paper we will show that Om is able to distinguish dynamical DDE from the cosmological constant in a robust manner both with and without reference to the value of the matter density, which can be a significant source of uncertainty for cosmological reconstruction. In other words, whether we know the current value of matter density or not, we can distinguish DDE model from LCDM even other dark energy models. The Om diagnostic is in many respects the logical companion to another geometric diagnostic–statefinder $r \equiv \frac{\ddot{a}}{aH^3}$ where $r = 1$ for LCDM while $r \neq 1$ for evolving DE models. Hence $r(z_1) - r(z_2)$ provides a null test for the cosmological constant. Similarly, the unevolving nature of $Om(z)$ in LCDM furnishes $Om(z_1) - Om(z_2)$ as a null test for the cosmological constant. Like the statefinder, Om depends only upon the expansion history of our Universe. However, while the statefinder r involves the third derivative of the expansion factor $a(t)$, Om depends upon its first derivative only. Therefore, Om is much easier to reconstruct from observations.

This paper is organized as follows: Basic equations of DDE model and introduction to Om diagnostic are firstly introduced in Sec.II. Based on these, we differentiate DDE model from LCDM and investigate the influence of coupling parameter α on the Om_{DDE} . These results are shown in figures mathematically. Sec.III is conclusions.

2. Om Diagnostic For DDE Model

Now let us consider the action of the Weyl-scaled induced gravitational theory:

$$S = \int d^4X \sqrt{-g} [\frac{1}{2}R(g_{\mu\nu}) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - W(\sigma) + L_{fluid}(\psi)] \quad (1)$$

where $L_{fluid}(\psi) = \frac{1}{2}g^{\mu\nu}e^{-\alpha\sigma}\partial_\mu\psi\partial_\nu\psi - e^{-2\alpha\sigma}V(\psi)$, $\alpha = \sqrt{\frac{\kappa^2}{2\varpi+3}}$ with ϖ being an important parameter in Weyl-scaled induced gravitational theory, σ is dilaton field, $W(\sigma)$ is dilaton scalar potential, $g_{\mu\nu}$ is the Pauli metric which can really represent the massless spin-two graviton and should be considered to be physical metric[11]. We work in units($\kappa^2 \equiv 8\pi G = 1$). When $W(\sigma) = 0$, Weyl-scaled induced gravitational theory will reduce to the Einstein-Brans-Dicke theory. We consider dilaton field as the candidate of DE and call Weyl-scaled induced gravitational theory as dilaton dark energy(DDE) model.

In Friedmann-Robertson-Walker universe, the field equations become:

$$H^2 = \frac{1}{3}[\rho_\sigma + e^{-\alpha\sigma}\rho_m] \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(e^{-\alpha\sigma}\rho_m + \rho_\sigma + p_\sigma) \quad (3)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dW(\sigma)}{d\sigma} = \frac{1}{2}\alpha e^{-\alpha\sigma}\rho_m \quad (4)$$

where $H = \frac{\dot{a}}{a}$ is Hubble parameter, ρ_σ and $\rho_m = \rho_{m0}\frac{e^{\frac{1}{2}\alpha\sigma}}{a^3}$ are dark energy density and matter energy density respectively. The effective energy density ρ_σ and the effective pressure p_σ of dilaton field can be expressed as follows

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + W(\sigma) \quad (5)$$

$$p_\sigma = \frac{1}{2}\dot{\sigma}^2 - W(\sigma) \quad (6)$$

We can rewrite Eq.2 as follows:

$$H^2 = H_0^2[(1 - \Omega_{m0})E(z) + \Omega_{m0}e^{-\frac{1}{2}\alpha\sigma}(1+z)^3] \quad (7)$$

where $\Omega_{m0} \equiv \rho_{m0}/3H_0^2$ is matter density parameter, and $E(z)$ is function of dark energy.

Now we introduce Om geometric diagnostic[12] which has been studied by many authors[13]

$$Om(x) \equiv \frac{h^2(x) - 1}{x^3 - 1}, \quad x = 1 + z, \quad h^2(x) = \frac{H^2}{H_0^2} \quad (8)$$

For dark energy with a constant equation of state $\omega = \text{const}$,

$$Om(x) = \Omega_{m0} + (1 - \Omega_{m0}) \frac{x^{3(1+\omega)} - 1}{x^3 - 1} \quad (9)$$

We can easily find

$$Om(x) = \Omega_{m0} \quad (10)$$

in LCDM, whereas $Om(x) > \Omega_{m0}$ in quintessence ($\alpha > 0$) while $Om(x) < \Omega_{m0}$ in quintessence ($\alpha < 0$). So, $Om(x) - \Omega_{m0} = 0$ if candidate of DE is cosmological constant.

In this paper, we consider a simple form of dark energy function $E(z) = x^{3(1+\omega_\sigma)}$, so Hubble parameter H can be expressed

$$H^2(x) = H_0^2 [(1 - \Omega_{m0})x^{3(1+\omega_\sigma)} + \Omega_{m0}e^{-\frac{1}{2}\alpha\sigma}x^3] \quad (11)$$

where $(1 - \Omega_{m0})x^{3(1+\omega_\sigma)} + \Omega_{m0}e^{-\frac{1}{2}\alpha\sigma}x^3 = h^2(x)$.

According to Eq.(8), we get the form of Om in DDE model

$$Om(x)_{DDE} = \frac{h^2(x) - 1}{x^3 - 1} = \frac{(1 - \Omega_{m0})x^{3(1+\omega_\sigma)} + \Omega_{m0}e^{-\frac{1}{2}\alpha\sigma}x^3 - 1}{x^3 - 1} \quad (12)$$

Comparing Eq.(9) and Eq.(12), we can see that coupling factor $e^{-\frac{1}{2}\alpha\sigma}$ between dilaton field and matter can affect the evolutive behavior of Om_{DDE} . When coupling parameter $\alpha = 0$, Eq.(12) reduces to Eq.(9). When $\alpha = 0$ and $\omega_\sigma = -1$, Eq.(12) reduces to Eq.(10). In other word, Om_{DDE} diagnostic can still provide a *null test* of LCDM when $\alpha = 0$ and $\omega_\sigma = -1$ in DDE model.

We need deduce the expression of dilaton field $\sigma(z)$ if we want to know the influence of coupling factor $e^{-\frac{1}{2}\alpha\sigma}$ on the Om . So, according to Eq.5, we have

$$\frac{1}{2}\dot{\sigma}^2 + [1 + (\sigma - A)^2]e^{-B\sigma} = \rho_{\sigma 0}z^{-3(1+\omega_\sigma)} \quad (13)$$

where we consider the dilaton scalar potential as the form $W(\sigma) = [1 + (\sigma - A)^2]e^{-B\sigma}$ with A and B being constant. Many authors believe that field with this kind of potential are predicted in the low energy limit of M-theory[14]. We can rewrite Eq.(13) as follows

$$\frac{1}{2}H^2(x)x^2\left(\frac{d\sigma}{dx}\right)^2 + [1 + (\sigma - A)^2]e^{-B\sigma} = \rho_{\sigma 0}x^{3(1+\omega_\sigma)} \quad (14)$$

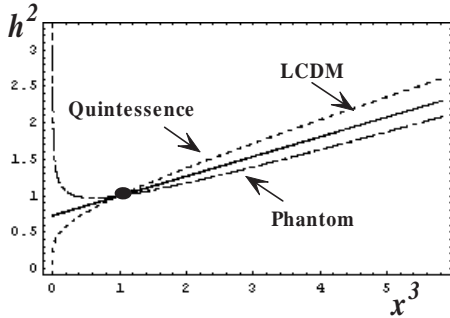


Fig.1 The evolutive trajectory of Hubble parameter square $h^2(x)$ with respect to x^3 (or $(1+z)^3$) for Quintessence(dot line), LCDM(real line) and Phantom(dot-dashed line). We set $\Omega_{m0} = 0.27$, $\alpha = 0.005$ and $\sigma_{z_0} = 0.3$.

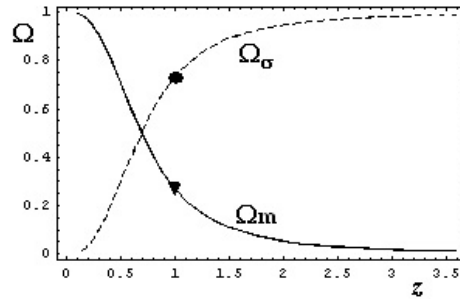


Fig.2 Ω_m and Ω_σ are plotted against the red-shift z for Quintessence. The filled Triangles and the dot denote the current value of matter energy density parameter Ω_{m0} and dark energy density parameter $\Omega_{\sigma 0}$.

From Fig.1, we can see that the trajectory of $h^2(x)$ with respect to x^3 for LCDM is always a straight line in the interval $-1 < z < 1.85$ whereas for Quintessence and Phantom the line is curved in the interval $-1 < z \ll 1$. Clearly, a comparison of Om at two different redshifts can lead to insights about the nature of DE even if the value of Om is not accurately known. Thus, the two-point difference diagnostic

$$Om(x_1, x_2) \equiv Om(x_1) - Om(x_2) \quad (15)$$

For LCDM

$$Om(x_1) = Om(x_2) \quad (16)$$

So, $Om(x_1, x_2) = 0$ if DE is a cosmological constant; $Om(x_1, x_2) > 0$ for quintessence while $Om(x_1, x_2) < 0$ for phantom.

The evolutive trajectories of matter energy density parameter Ω_m and Ω_σ are shown in Fig.2. The current value of matter energy density parameter Ω_{m0} and dark energy density parameter $\Omega_{\sigma0}$ are respective 0.27 and 0.73. We can see that the DDE will evolve into de Sitter space-time at late time. This result consists with the conclusion of our previous paper[15].

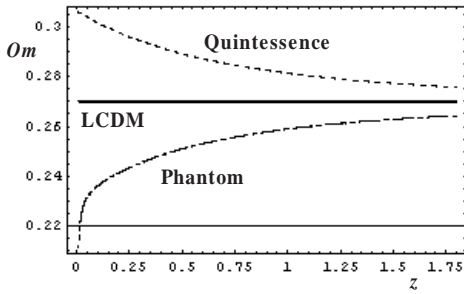


Fig.3 The evolutive trajectories of Om with respect to z for Quintessence(dot line), Phantom(dot-dashed line) and LCDM(real line). We set $\alpha = 0.05$, $\Omega_{m0} = 0.27$.

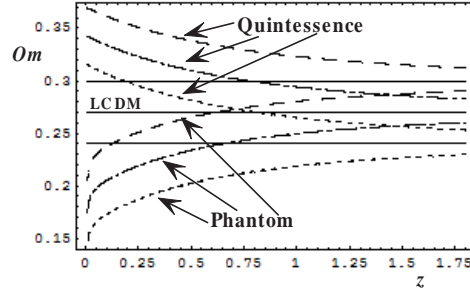


Fig.4 The evolutive trajectories of Om with respect to z for Quintessence, Phantom and LCDM(real line) when we set three different current values of $\Omega_{m0}=0.27$ (dot-dashed line), 0.24 (dot line), 0.30 (dashed line). We set $\alpha = 0.05$.

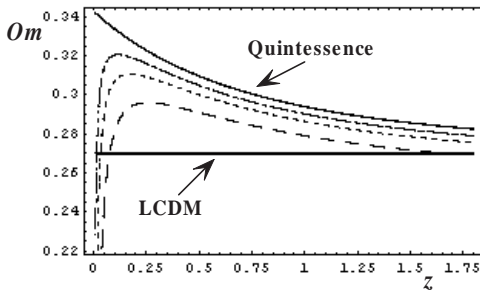


Fig.5 The evolutive trajectories of Om with respect to z for Quintessence when we set $\alpha=0.000005$ (real line), 0.05 (dot-dashed line), 0.1 (dot line) and 0.2 (dashed line). The horizontal line corresponds to LCDM.

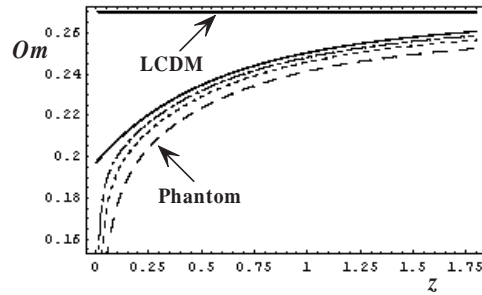


Fig.6 The evolutive trajectories of Om with respect to z for Phantom, when we set $\alpha=0.000005$ (real line), 0.05 (dot-dashed line), 0.1 (dot line) and 0.2 (dashed line). The horizontal line corresponds to LCDM.

According to Fig.3, we can see clearly that the evolutive behaviors of Om with respect to z for Quintessence, Phantom and LCDM are very different. In the interval $0 < z < 1.85$, the slope of Om for Quintessence is negative while the slope of Om for Phantom is positive. The horizontal straight line corresponds to LCDM. Therefore, we can easily distinguish Quintessence and Phantom model from LCDM by the trajectories of their Om (slope of Om). Furthermore, Fig.4 shows that the trajectories of Om for Quintessence, Phantom and LCDM when we set different values $\Omega_{m0}=0.27, 0.24$ and 0.32 . Clearly, whether we set a correct current

value of $\Omega_{m0}=0.27$ or incorrect values of $\Omega_{m0}=0.24$ and 0.3 , we can differentiate Quintessence and Phantom from LCDM. The trajectories of Om for Quintessence, Phantom and LCDM_{like} when $\Omega_{m0}=0.24$ and 0.3 can be regarded as the results that Quintessence, Phantom and LCDM($\Omega_{m0} = 0.27$) moves up($\Omega_{m0} = 0.3$) and down($\Omega_{m0} = 0.24$) paralleled. So, the slope of Om can differentiate between different models including Quintessence, Phantom and LCDM, even if the value of the matter density is not accurately known.

Fig.5 and Fig.6 show the influence of coupling parameter α on the Om diagnostic for Quintessence and Phantom respectively. In Fig.5, we can see that the trajectory of Om for Quintessence moves upward with decrease of α from 0.2 to 0.00005 . So, the asymptote of trajectory of Om with $\alpha \rightarrow 0$ is the horizontal straight line which corresponds to LCDM. Similarly, when the coupling parameter α changes from 0.2 to 0.00005 , the trajectory of Om for Phantom moves upward too. When the the couple parameter $\alpha \rightarrow 0$, LCDM is the the asymptote of trajectory of Om for Phantom.

3. Conclusions

In this paper we apply the Om diagnostic to DDE model. We have demonstrated that the plot of $h^2(x)-x^3$ for Quintessence($\omega_\sigma > -1$), Phantom($\omega_\sigma < -1$) and LCDM($\omega_\sigma = -1$). We can obtain that DDE model will evolve into de Sitter space-time for Quintessence($\omega_\sigma > -1$) at late time or "Big Rip" future singularity for Phantom($\omega_\sigma < -1$) as $z \rightarrow 0(x \rightarrow 1)$. Fig.2 also shows DDE model in Quintessence admits a late time de Sitter attractor solution. According to the expression $\frac{Om(z)-\Omega_{m0}}{1-\Omega_{m0}} \simeq 1 + \omega_{\sigma 0} + \frac{\Omega_{m0}(e^{-\frac{1}{2}\alpha\sigma_0}-1)}{1-\Omega_{m0}}$ when $x \rightarrow 1(z \rightarrow 0)$ and the numerical results of differential equation Eq.(14), it is can be found that current EOS of DDE $\omega_{\sigma 0} \simeq -0.952$ which fits the combination WMAP5+BAO+SN($\omega = -0.992 \pm_{0.062}^{0.061}$) well.

We have also plotted the trajectory of Om with respect to z for Quintessence, Phantom and LCDM. We can easily distinguish DDE model from LCDM according to the slope of their evolutive trajectories. Horizontal straight line corresponds to LCDM while the slope of Om for Quintessence is negative and the slope of Om for Phantom is positive. The detailed numerical investigations show that we can differentiate DDE models from LCDM even if the value of the matter density is not accurately known.

At last, we investigate the influence of coupling parameter α on Om for DDE model. With the decrease of α , the shape of trajectory of Om with respect to z for both Quintessence and Phantom, becomes more and more close to LCDM which is the asymptote of trajectory of Om with $\alpha \rightarrow 0$. The recent observational limit to coupling parameter α is $\alpha < 0.001$ [16]. According to the numerical results, the influence of coupling parameter α on Om is very tiny in observational range, which consists with our previous results.

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